The confluent hypergeometric function can now be expressed as:

$$F(l + 1 - n,\ 2l + 2,\ 2rn^{-1}) = \sum_{\nu=0}^{\infty} n^{-\nu}\, \Phi_{l,\nu}(r), \tag{29}$$

with

$$\Phi_{l,\nu} = (-1)^{\nu} (2l + 1)! \sum_{k=\nu}^{\infty} \frac{(-1)^k\, a_{\nu}^{k,l}\, (2r)^k}{k!\,(2l + 1 + k)!}. \tag{30}$$

The functions $\Phi_{l,\nu}$, can be transformed to sums of Bessel functions, when the coefficients $a_{\nu}^{k,l}$ are written in a convenient form. It follows then that the first functions $\Phi_{l,\nu}$ are, when the abbreviation $z = 2\sqrt{2r}$ is used:

$$\Phi_{l,0}(z) = (2l + 1)!\ (\tfrac{1}{2}z)^{-2l-1} J_{2l+1}(z), \tag{31}$$

$$\Phi_{l,1}(z) = \tfrac{1}{2}(2l + 1)!\ (\tfrac{1}{2}z)^{-2l+1} J_{2l+1}(z), \tag{32}$$

$$\Phi_{l,2}(z) = \tfrac{1}{24}(2l + 1)!\ [\{ (\tfrac{1}{2}x)^{-2l-1} (8l^3 + 12l^2 + 4l) + (\tfrac{1}{2}z)^{-2l+1}(2l + 2) + 3(\tfrac{1}{2}z)^{-2l+3}\} J_{2l+1}(z) + \{-(\tfrac{1}{2}z)^{-2l} (4l^2 + 4l) - 2(\tfrac{1}{2}z)^{-2l+2}\} J_{2l}(z)]. \tag{33}$$

To obtain these expressions, $a_{\nu}^{k,l}/k!$ should be written down as a sum of reciprocal factorials; so is e.g. the form

$$\frac{a_2^{k,l}}{k!} = \frac{{}^1/_2 l^2 + {}^3/_2 l + 1}{(k-2)!} + \frac{{}^1/_2 l + {}^5/_6}{(k-3)!} + \frac{{}^1/_8}{(k-4)!} \tag{34}$$

appropriate. After that, recurrence formulae for the Bessel functions should be applied.

Of special interest for the electronic levels, studied is this note, are the cases $l = 0$:

$$\Phi_{0,0}(z) = ({}^1/_2 z)^{-1} J_1(z), \tag{35}$$

$$\Phi_{0,1}(z) = {}^1/_4 z\, J_1(z), \tag{36}$$

$$\Phi_{0,2}(z) = \{{}^1/_{12}({}^1/_2 z) + {}^1/_8({}^1/_2 z)^3\} J_1(z) - {}^1/_{12}({}^1/_2 z)^2 J_0(z), \tag{37}$$

and $l = 1$:

$$\Phi_{1,0}(z) = 6({}^1/_2 z)^{-3} J_3(z), \tag{38}$$

$$\Phi_{1,1}(z) = 3({}^1/_2 z)^{-1} J_3(z), \tag{39}$$

$$\Phi_{1,2}(z) = \{6({}^1/_2 z)^{-3} + ({}^1/_2 z)^{-1} + {}^3/_4({}^1/_2 z)\} J_3(z) + \{-2({}^1/_2 z)^{-2} - {}^1/_2\} J_2(z). \tag{40}$$

To find the character of the (E, r_0)-curve in the neighbourhood of $E = 0$ or $n^{-1} = 0$ it is necessary to consider the nodes r_0 of F or, by way of approximation, of a certain number of terms of the development (29) When we take:

$$\Phi_{l,0} + n^{-1}\, \Phi_{l,1} + n^{-2}\, \Phi_{l,2} = 0, \tag{41}$$

and put

$$r_0 = r_{00} + r_{01} + r_{02}, \tag{42}$$

where r_{00} is of zeroth order and r_{01} and r_{02} of first and second order in n^{-1}, it is found after expanding the function Φ in Taylor series and equating terms of equal order [2]):